

Complete electroweak $\mathcal{O}(\alpha)$ corrections to charged-current $e^+e^- \rightarrow 4$ fermion processes

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Abstract:

The complete electroweak $\mathcal{O}(\alpha)$ corrections are calculated for the charged-current four-fermion production processes $e^+e^- \rightarrow \nu_\tau\tau^+\mu^-\bar{\nu}_\mu$, $u\bar{d}\mu^-\bar{\nu}_\mu$, and $u\bar{d}s\bar{c}$. The calculation is performed using complex gauge-boson masses, supplemented by complex couplings to restore gauge invariance. The evaluation of the occurring one-loop tensor integrals, which include 5- and 6-point functions, requires new techniques. Explicit numerical results are presented for total cross sections in the energy range from the W-pair-production threshold region up to a scattering energy of 2 TeV. A comparison with the predictions based on the “double-pole approximation” (DPA) provided by the generator RACOONWW reveals corrections beyond DPA of $\lesssim 0.5\%$ in the energy range 170–300 GeV, in agreement with previous estimates for the intrinsic DPA uncertainty. The difference to the DPA increases to 1–2% for $\sqrt{s} \sim 1\text{--}2$ TeV. At threshold, where the DPA becomes unreliable, the full $\mathcal{O}(\alpha)$ calculation corrects an improved Born approximation (IBA) by about 1.6%, also consistent with an error estimate of the IBA.

1 Introduction

At LEP2, W-pair-mediated four-fermion ($4f$) production was experimentally explored with quite high precision (see Ref. [1] and references therein). The total W-pair cross section was measured from threshold up to a centre-of-mass (CM) energy of 207 GeV; combining the cross-section measurements a precision of $\sim 1\%$ was reached. The W-boson mass M_W was determined from the threshold cross section with an error of ~ 200 MeV and by reconstructing the W bosons from their decay products within ~ 40 MeV. Deviations from the Standard Model (SM) triple gauge-boson couplings, usually quantified in the parameters Δg_1^Z , $\Delta \kappa_\gamma$, and λ_γ , were constrained within a few per cent.

The LEP2 measurements had set the scale in accuracy in the theoretical predictions for W-pair-mediated $4f$ production. The theoretical treatment and the gained level in precision are reviewed in Refs. [2, 3]. The W bosons are treated as resonances in the full $4f$ processes, $e^+e^- \rightarrow 4f (+\gamma)$. Radiative corrections are split into universal and non-universal corrections. The former comprise leading-logarithmic corrections from initial-state radiation (ISR), higher-order corrections included by using appropriate effective couplings, and the Coulomb singularity. These corrections can be combined with the lowest-order matrix elements easily. The remaining corrections are called non-universal, since they depend on the process under investigation. For LEP2 accuracy, it was sufficient to include these corrections in the so-called double-pole approximation (DPA), where only the leading term in an expansion about the poles in the two W-boson propagators is taken into account. Different versions of such a DPA have been used in the literature [4, 5, 6, 7, 8]. Although several Monte Carlo programs exist that include universal corrections, only two event generators, YFSWW [5, 6] and RACOONWW [7, 9, 10], include non-universal corrections.

In the DPA approach, the W-pair cross section can be predicted within $\sim 0.5\%$ (0.7%) in the energy range between 180 GeV (170 GeV) and ~ 500 GeV, which was sufficient for the LEP2 accuracy of $\sim 1\%$ for energies 170–207 GeV. In the threshold region ($\sqrt{s} \lesssim 170$ GeV), the DPA is not reliable, and the best available prediction results from an improved Born approximation (IBA) based on leading universal corrections only, and thus possesses an intrinsic uncertainty of $\sim 2\%$. At energies above 500 GeV effects beyond $\mathcal{O}(\alpha)$, such as Sudakov logarithms at higher orders, become important and should be included in predictions at per-cent accuracy.

At a future International e^+e^- Linear Collider (ILC) [11, 12, 13], the accuracy of the cross-section measurement will be at the per-mille level, and the precision of the W-mass determination is expected to be ~ 10 MeV [14] by direct reconstruction and ~ 7 MeV from a threshold scan of the total W-pair-production cross section [11, 12]. The theoretical uncertainty (TU) for the direct mass reconstruction at LEP2 is estimated to be of the order of ~ 5 MeV [15] to $\lesssim 10$ MeV [16], based on results of YFSWW and RACOONWW; theoretical improvements are, thus, desirable for an ILC. For the cross-section prediction at threshold the TU is only $\sim 2\%$, because it is based on an IBA, and thus is definitely insufficient for the planned precision measurement of M_W in a threshold scan. The main sensitivity of all observables to anomalous couplings in the triple gauge-boson vertices is provided by the W-pair production angle distribution. The TU in constraining the parameter λ_γ was estimated to be ~ 0.005 [17] for the LEP2 analysis. Since a future ILC

is more sensitive to anomalous gauge-boson couplings than LEP2 by more than an order of magnitude, a further reduction of the uncertainties resulting from missing radiative corrections is necessary. In summary, these considerations demonstrate the necessity of a full one-loop calculation for the $e^+e^- \rightarrow 4f$ process and of further improvements by leading higher-order corrections.

In this paper we present first results of a complete $\mathcal{O}(\alpha)$ calculation (improved by higher-order ISR) for the $4f$ final states $\nu_\tau\tau^+\mu^-\bar{\nu}_\mu$, $u\bar{d}\mu^-\bar{\nu}_\mu$, and $u\bar{d}s\bar{c}$, which are relevant for W-pair production.¹ The actual calculation is rather complicated.² Technically the occurring one-loop tensor integrals comprise 5- and 6-point functions up to rank 3, and conceptually the W-boson resonances require a treatment in loop diagrams that preserves gauge invariance. Details of our approach will be described in a forthcoming publication; in the next section we just sketch the most important features of the calculation. In Section 3 we present explicit numerical results on total cross sections for scattering energies from near the W-pair-production threshold up to 2 TeV. Particular attention is paid to a comparison with the DPA and IBA approaches used at LEP2.

2 Method of calculation

The actual calculation builds upon the RACOONWW approach [7], where real-photonic corrections are based on full matrix elements and virtual corrections are treated in DPA. Real and virtual corrections are combined either using two-cutoff phase-space slicing or employing the dipole subtraction method [20, 21] for photon radiation. We also include leading-logarithmic initial-state radiation (ISR) beyond $\mathcal{O}(\alpha)$ in the structure-function approach (Ref. [2] and references therein). The presented calculation only differs in the treatment of the virtual corrections from RACOONWW. We neglect the masses of the fermions whenever possible, i.e. everywhere but in the mass-singular logarithms, and set the quark-mixing matrix to the unit matrix.

In the following we sketch the main difficulties in the calculation and briefly explain our solutions.

2.1 Technical issues

In contrast to the DPA approach, the one-loop calculation of an $e^+e^- \rightarrow 4f$ process requires the evaluation of 5- and 6-point one-loop tensor integrals. Some 6-point diagrams are shown in Figure 1 for illustration. For the generic $f_1\bar{f}_2f_3\bar{f}_4$ final state, where f_1 and f_3 are different fermions excluding electrons and electron neutrinos and f_2 and f_4 their isospin partners, there are 40 hexagon diagrams, 112 pentagon diagrams, and 227 (220) box diagrams in the conventional 't Hooft–Feynman gauge (background-field gauge). We calculate the 6-point integrals by directly reducing them to six 5-point

¹Electrons and/or positrons in the final state are not yet considered; they deserve further refinements, in particular the inclusion of finite-electron-mass effects in the domain of forward-scattered e^\pm . We also do not yet include final states that can also be produced via two resonant Z bosons, so called mixed CC/NC reactions. These can be taken into account in lowest order in RacoOnWW.

²Some of the problems appearing in a first attempt of such a calculation were already described in Ref. [18]. Recently the authors of the GRACE/1-LOOP system reported on progress towards a full one-loop calculation for $e^+e^- \rightarrow \mu^-\bar{\nu}_\mu u\bar{d}$ in Ref. [19] so that one can expect that the system will be able to deal with $e^+e^- \rightarrow 4f$ processes at one loop in the near future.

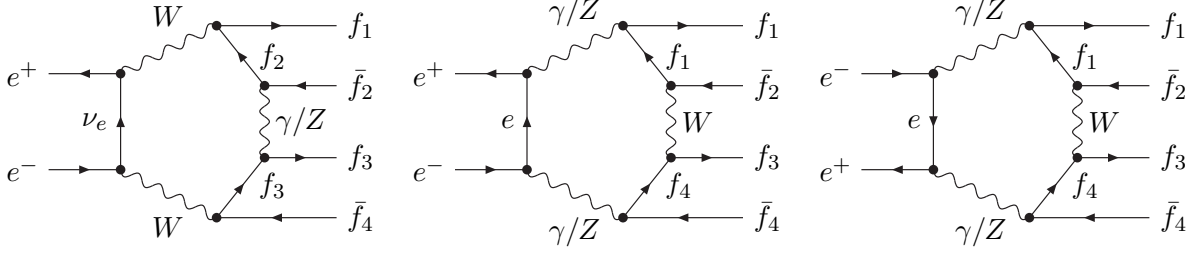


Figure 1: Ten 6-point diagrams contributing to $e^+e^- \rightarrow f_1\bar{f}_2f_3\bar{f}_4$. The remaining 30 6-point diagrams are obtained by reversing the fermion flow in one or both of the fermion chains corresponding to the outgoing fermions.

functions, as described in Refs. [22, 23]. The 5-point integrals are reduced to five 4-point functions following the method of Ref. [24]. These reduction steps involve so-called (modified) Cayley determinants, the zeroes of which are related to the Landau singularities of the (sub-)diagrams. We did not encounter numerical problems with these determinants. Note that this reduction of 5- and 6-point integrals to 4-point integrals does not involve inverse Gram determinants composed of external momenta, which naturally occur in the Passarino–Veltman reduction [25] of tensor to scalar integrals. The latter procedure leads to serious numerical problems when the Gram determinants become small, which happens usually near the boundary of phase space but can also occur within phase space because of the indefinite Minkowski metric.

We use, however, Passarino–Veltman reduction to calculate tensor integrals up to 4-point functions, which involves inverse Gram determinants built from two or three momenta. This, in fact, leads to numerical instabilities in phase-space regions where these Gram determinants become small. For these regions we have worked out two “rescue systems”: one makes use of expansions of the tensor coefficients about the limit of vanishing Gram determinants; in the other, alternative method we numerically evaluate a specific tensor coefficient, the integrand of which is logarithmic in Feynman parametrization, and derive the remaining coefficients as well as the scalar integral from it algebraically. This reduction again involves only inverse Cayley determinants, but no inverse Gram determinants.

In addition to the evaluation of the one-loop integrals, also the evaluation of the three spinor chains corresponding to the three external fermion–antifermion pairs is non-trivial, because the chains are contracted with each other and/or with four-momenta in many different ways. There are $\mathcal{O}(10^3)$ different chains to calculate, so that an algebraic reduction to a standard form which involves only very few standard chains is desirable. We have worked out algorithms that reduce all occurring spinor chains to $\mathcal{O}(10)$ standard structures without introducing coefficients that lead to numerical problems.

2.2 Conceptual issues

The description of resonances in (standard) perturbation theory requires a Dyson summation of self-energy insertions in the resonant propagator in order to introduce the imaginary part provided by the finite decay width into the propagator denominator. It is well known that this procedure in general violates gauge invariance, i.e. destroys Slavnov–

Taylor or Ward identities and disturbs the cancellation of gauge-parameter dependences, because different perturbative orders are mixed (see, for instance, Ref. [3] and references therein). Several solutions have been described for lowest-order predictions, but the general problem is still considered as unsolved for a consistent evaluation of radiative corrections. The DPA provides a gauge-invariant answer in terms of an expansion about the resonance,³ but in the full calculation we are after a unified description that is valid both for resonant and non-resonant regions in phase space, without any matching between different treatments for different regions.

In our calculation we use a generalization of the so-called “complex-mass scheme” (CMS), which was introduced in Ref. [9] for lowest-order calculations, to the one-loop level. In this approach the W- and Z-boson masses are consistently considered as complex quantities, defined as the locations of the poles in the complex p^2 plane of the corresponding propagators with momentum p . Gauge invariance is preserved if the complex masses are introduced everywhere in the Feynman rules, in particular, in the definition of the weak mixing angle,

$$c_w^2 = 1 - s_w^2 = \frac{M_W^2 - iM_W\Gamma_W}{M_Z^2 - iM_Z\Gamma_Z}, \quad (2.1)$$

which is now derived from the ratio of the complex mass squares. The (algebraic) relations, such as Ward identities, that follow from gauge invariance remain valid, because the gauge-boson masses are modified only by an analytic continuation. Since this continuation already modifies the lowest-order predictions by changing the gauge-boson masses, double-counting of higher-order effects has to be carefully avoided by an appropriate renormalization procedure.

The use of complex gauge-boson masses necessitates the consistent use of these complex masses also in loop integrals. To this end, we have derived all relevant one-loop integrals with complex internal masses. The IR-singular integrals were taken from Ref. [27]. Concerning the non-IR singular cases, we have analytically continued the results of Ref. [28] for the 2-point and 3-point functions⁴, and the relevant results of Ref. [29] for the 4-point functions. We have checked all these results by independent direct calculation of the Feynman-parameter integrals.

2.3 Checks

The complexity of the calculation enforces a number of consistency checks to prove the reliability of the results. We have performed the following checks:

- *UV finiteness* is checked by verifying the independence of the (arbitrary) reference scale μ of dimensional regularization.
- *IR finiteness* is checked by varying the logarithm $\ln \lambda$ of the (formally infinitesimal) photon mass λ , which leaves the sum of the virtual and the soft-photonic corrections

³The recently proposed approach to describe unstable particles within an effective field theory [26] is equivalent to a pole expansion.

⁴Note that the result of Ref. [28] for the scalar two-point function is not valid in general for complex masses. In this case an extra η function has to be added. The same comment applies to the results for the 2-point tensor integrals in Ref. [25].

(slicing approach) or of the virtual and endpoint contributions (i.e. the singular parts that are subtracted from the virtual corrections in the subtraction approach) invariant.

- *Mass singularities* related to collinear photon emission or exchange are checked by verifying the independence of the sum of the virtual corrections and the subtraction endpoint contributions from the small masses of the external fermions.
- *Gauge invariance* is checked by comparing the result obtained within the 't Hooft–Feynman gauge with an independent result obtained within the background-field formalism [30]. Apart from diagrams that involve only fermion–gauge-boson couplings in the loops, the contributions of individual Feynman graphs are in general different in the two approaches.
- *The real corrections* are taken over from RACOONWW [7, 9], where they were checked by two independent calculations in detail. Moreover, agreement was found between the results obtained with phase-space slicing or dipole subtraction.
- *The scalar loop integrals* for complex masses have been calculated in two completely independent ways and implemented in two independent in-house libraries. We checked that these results agree with each other and in the limit of zero width also with FF [31].
- *Two completely independent* calculations have been performed within our group, revealing good agreement. All algebraic manipulations, including the generation of Feynman diagrams and the reduction of amplitudes to standard forms, have been done using two independent programs.⁵ The evaluations of all scalar and tensor loop integrals are based on two independent in-house libraries, which employ the two different rescue systems mentioned above. We consider this as the most important and convincing check.

We emphasize that all these checks, including the gauge-invariance check, have been carried out for non-zero gauge-boson widths.

⁵The amplitudes are generated with FEYNARTS, using the two independent versions 1 and 3, as described in Refs. [32] and [33], respectively. The algebraic manipulations are performed using two independent in-house programs implemented in MATHEMATICA, one of which builds upon FORMCALC [34].

3 Numerical results

3.1 Input parameters and setup

For the numerical evaluation we use the following set of SM parameters,

$$\begin{aligned}
G_\mu &= 1.16637 \times 10^{-5} \text{ GeV}^{-2}, & \alpha(0) &= 1/137.03599911, & \alpha_s &= 0.1187, \\
M_W &= 80.425 \text{ GeV}, & M_Z &= 91.1876 \text{ GeV}, & \Gamma_Z &= 2.4952 \text{ GeV}, \\
M_H &= 115 \text{ GeV}, \\
m_e &= 0.51099892 \text{ MeV}, & m_\mu &= 105.658369 \text{ MeV}, & m_\tau &= 1.77699 \text{ GeV}, \\
m_u &= 66 \text{ MeV}, & m_c &= 1.2 \text{ GeV}, & m_t &= 178 \text{ GeV}, \\
m_d &= 66 \text{ MeV}, & m_s &= 150 \text{ MeV}, & m_b &= 4.3 \text{ GeV},
\end{aligned} \tag{3.1}$$

which essentially follows Ref. [35]. For the top-quark mass m_t we have taken the more recent value of Ref. [36]. The masses of the light quarks are adjusted to reproduce the hadronic contribution to the photonic vacuum polarization of Ref. [37]. Since we parametrize the lowest-order cross section with the Fermi constant G_μ (G_μ scheme), i.e. we derive the electromagnetic coupling α according to $\alpha_{G_\mu} = \sqrt{2}G_\mu M_W^2(1 - M_W^2/M_Z^2)/\pi$, the results are practically independent of the masses of the light quarks. Moreover, this procedure absorbs the corrections proportional to m_t^2/M_W^2 in the fermion–W-boson couplings and the running of $\alpha(Q^2)$ from $Q^2 = 0$ to the electroweak scale. In the relative radiative corrections, we use, however, $\alpha(0)$ as coupling parameter, which is the correct effective coupling for real photon emission.

QCD corrections are treated in the “naive” approach of multiplying cross sections and partial decay rates by factors $(1 + \alpha_s/\pi)$ per hadronically decaying W boson. The W-boson width Γ_W is calculated from the above input including electroweak $\mathcal{O}(\alpha)$ and QCD corrections, yielding

$$\Gamma_W = 2.09269848 \dots \text{ GeV}. \tag{3.2}$$

For the full one-loop calculation and for the DPA approach this procedure ensures that the effective branching ratios for the leptonic, semileptonic, and hadronic W decays, which result from the integration over the decay fermions, add up to 1. In order to keep this normalization also for the IBA, Γ_W is calculated in the corresponding approximation, i.e. in the G_μ scheme without electroweak corrections, yielding

$$\Gamma_W|_{\text{IBA}} = 2.10009936 \dots \text{ GeV}. \tag{3.3}$$

A detailed description of the IBA, as used in RACOONWW, can be found in Ref. [10].

In the following we discuss only total cross sections without any phase-space cuts. The presented results have been obtained with 10^7 events, using the subtraction method.

3.2 Results for total cross sections

Tables 1–3 show some representative results on total cross sections for the final states $\nu_\tau \tau^+ \mu^- \bar{\nu}_\mu$, $u \bar{d} \mu^- \bar{\nu}_\mu$, and $u \bar{d} s \bar{c}$ in various approximations for different CM energies \sqrt{s} . The numbers in parentheses represent the uncertainties in the last digits of the predictions.

$e^+e^- \rightarrow \nu_\tau \tau^+ \mu^- \bar{\nu}_\mu$					
\sqrt{s}/GeV	Born(FW)	Born(CMS)	IBA	DPA	ee4f
161	50.04(2)	50.01(2) [−0.06%]	37.18(2) [−25.67(6)%]	37.08(2) [−25.90(3)%]	37.95(2) [−24.12(4)%]
170	160.53(6)	160.44(6) [−0.06%]	129.12(6) [−19.52(5)%]	129.17(6) [−19.53(3)%]	129.23(6) [−19.45(3)%]
189	216.57(8)	216.45(8) [−0.06%]	191.89(8) [−11.35(5)%]	191.66(9) [−11.50(2)%]	190.89(9) [−11.81(3)%]
200	220.41(9)	220.29(9) [−0.06%]	201.13(9) [−8.70(6)%]	200.04(10) [−9.24(2)%]	199.21(10) [−9.57(3)%]
500	86.95(5)	86.90(5) [−0.06%]	92.79(5) [+6.78(9)%]	89.81(6) [+3.29(3)%]	89.13(6) [+2.57(4)%]
1000	33.35(2)	33.33(2) [−0.06%]	38.04(4) [+14.12(14)%]	35.76(3) [+7.21(5)%]	35.37(3) [+6.12(6)%]

Table 1: Total cross sections in fb for $e^+e^- \rightarrow \nu_\tau \tau^+ \mu^- \bar{\nu}_\mu$ in Born approximation (in the fixed-width and complex-mass schemes), IBA, DPA, and using the full $\mathcal{O}(\alpha)$ correction (ee4f); all but the Born cross sections include higher-order ISR corrections.

$e^+e^- \rightarrow u\bar{d}\mu^- \bar{\nu}_\mu$					
\sqrt{s}/GeV	Born(FW)	Born(CMS)	IBA	DPA	ee4f
161	150.15(7)	150.07(7) [−0.06%]	115.75(7) [−22.87(6)%]	115.48(7) [−23.09(3)%]	118.12(8) [−21.29(4)%]
170	481.6(2)	481.3(2) [−0.06%]	402.0(2) [−16.48(5)%]	401.8(2) [−16.58(3)%]	402.1(2) [−16.47(3)%]
189	649.7(3)	649.4(3) [−0.06%]	597.4(3) [−8.00(5)%]	596.1(3) [−8.26(3)%]	593.8(3) [−8.55(3)%]
200	661.3(3)	660.9(3) [−0.06%]	626.2(3) [−5.26(6)%]	622.2(3) [−5.91(3)%]	619.7(3) [−6.24(3)%]
500	260.9(1)	260.8(1) [−0.06%]	288.9(2) [+10.78(9)%]	279.6(2) [+7.14(4)%]	277.4(2) [+6.37(4)%]
1000	100.10(6)	100.04(6) [−0.06%]	118.44(13) [+18.39(15)%]	111.45(9) [+11.34(5)%]	110.17(10) [+10.13(7)%]

Table 2: Total cross sections in fb for $e^+e^- \rightarrow u\bar{d}\mu^- \bar{\nu}_\mu$ in Born approximation (in the fixed-width and complex-mass schemes), IBA, DPA, and using the full $\mathcal{O}(\alpha)$ correction (ee4f); all but the Born cross sections include higher-order ISR and QCD corrections.

$$e^+e^- \rightarrow u\bar{d}s\bar{c}$$

\sqrt{s}/GeV	Born(FW)	Born(CMS)	IBA	DPA	ee4f
161	450.5(2)	450.3(2) [−0.06%]	360.4(2) [−19.97(6)%]	359.7(2) [−20.16(4)%]	367.6(3) [−18.37(4)%]
170	1444.9(5)	1444.1(5) [−0.06%]	1251.6(5) [−13.33(5)%]	1250.1(6) [−13.49(3)%]	1250.7(6) [−13.39(3)%]
189	1949.3(8)	1948.2(8) [−0.06%]	1860.0(8) [−4.53(6)%]	1853.8(9) [−4.90(3)%]	1846.9(9) [−5.20(3)%]
200	1983.9(8)	1982.9(8) [−0.06%]	1949.5(9) [−1.68(6)%]	1935.3(9) [−2.45(3)%]	1927.7(9) [−2.78(3)%]
500	782.9(4)	782.5(4) [−0.06%]	899.4(5) [+14.94(9)%]	869.4(6) [+11.05(4)%]	862.9(6) [+10.27(4)%]
1000	300.3(2)	300.1(2) [−0.06%]	368.7(4) [+22.86(16)%]	346.1(3) [+15.26(5)%]	342.1(3) [+13.98(7)%]

Table 3: As in Table 2, but for the process $e^+e^- \rightarrow u\bar{d}s\bar{c}$.

Columns two and three in each table contain the two versions of the lowest-order cross section for the full $e^+e^- \rightarrow 4f$ processes corresponding to the different treatments of finite-width effects as provided by the “fixed-width scheme” (FW) and the complex-mass scheme (CMS). In the FW scheme the finite constant width, and thus the complex mass, is only inserted into the resonant propagators. The relative difference $\sigma_{\text{Born}}(\text{CMS})/\sigma_{\text{Born}}(\text{FW}) - 1$ of the schemes in lowest order is given by the numbers in square brackets in the third columns. Note that the difference is only 0.06% for the considered energies, so that it is not essential to which Born cross section we normalize relative corrections. We have not given an error on this difference, because the two Born predictions are strongly correlated. The fourth columns show the IBA [10] implemented in RACOONWW, which is based on universal corrections only and includes solely the contributions of the CC03 diagrams; the numbers in square brackets are defined as $\delta_{\text{IBA}} = \sigma_{\text{IBA}}/\sigma_{\text{Born}}(\text{CMS}) - 1$. The fifth columns show the DPA of RACOONWW⁶, which comprises also non-universal corrections [7]; the numbers in square brackets are defined as $\delta_{\text{DPA}} = \sigma_{\text{DPA}}/\sigma_{\text{Born}}(\text{FW}) - 1$. We normalize σ_{DPA} to $\sigma_{\text{Born}}(\text{FW})$, because the lowest-order part of the DPA is per default evaluated in the FW scheme in RACOONWW. Finally, the last columns (ee4f) contain the full one-loop corrections to $e^+e^- \rightarrow 4f$; the numbers in square brackets are defined as $\delta_{\text{ee4f}} = \sigma_{\text{ee4f}}/\sigma_{\text{Born}}(\text{CMS}) - 1$. Here we normalize to $\sigma_{\text{Born}}(\text{CMS})$, because the full $e^+e^- \rightarrow 4f$ calculation is consistently performed in the CMS. Note that all but the “Born” numbers include improvements by ISR effects beyond $\mathcal{O}(\alpha)$, as described in Ref. [7]. Additionally the results on the semileptonic and hadronic cross sections (all but the Born cross results) include naive QCD corrections, as explained in Section 3.1. For better

⁶We recall that the DPA of RACOONWW goes beyond a pure pole approximation in two respects. The real-photon corrections are based on the full $e^+e^- \rightarrow 4f + \gamma$ matrix elements, and the Coulomb singularity is included for off-shell W bosons. Further details can be found in Ref. [7].

illustration Figure 2 depicts the predictions for the energy ranges of LEP2 and of the high-energy phase of a future ILC, focusing on the leptonic final state $\nu_\tau\tau^+\mu^-\bar{\nu}_\mu$. The respective figures for the relative corrections δ to the semileptonic and hadronic final states look almost identical, up to an offset resulting from the QCD corrections.

A comparison between the DPA and the predictions based on the full $\mathcal{O}(\alpha)$ corrections reveals differences in the relative corrections δ of $\lesssim 0.5\%$ (0.7%) for CM energies ranging from $\sqrt{s} \sim 170$ GeV to 300 GeV (500 GeV). This is in agreement with the expected reliability of the DPA, as discussed in Refs. [3, 6, 7]. At higher energies, the deviations increase and reach 1–2% at $\sqrt{s} = 1\text{--}2$ TeV. In the threshold region ($\sqrt{s} \lesssim 170$ GeV), as expected, the DPA also becomes worse w.r.t. the full one-loop calculation, because the naive error estimate of $(\alpha/\pi) \times (\Gamma_W/M_W)$ times some numerical safety factor of $\mathcal{O}(1\text{--}10)$ for the corrections missing in the DPA has to be replaced by $(\alpha/\pi) \times \Gamma_W/(\sqrt{s} - 2M_W)$ in the threshold region and thus becomes large. In view of that, the DPA is even surprisingly good near threshold. For CM energies below 170 GeV the LEP2 cross section analysis was based on approximations like the shown IBA, which follows the full one-loop corrections even below the threshold at $\sqrt{s} = 2M_W$ within an accuracy of about 2%, as expected in Ref. [10]. The shape of the relative corrections in the threshold region is determined by ISR. The minimum in the relative corrections is correlated to the maximum in the slope of the total cross section.

A detailed comparison [3] between the DPA versions of Ref. [4], of YFSWW [5, 6], and of RACOONWW [7, 9, 10] showed differences of the same order (or smaller) as the differences between the DPA of RACOONWW and the result based on the full one-loop calculation of $e^+e^- \rightarrow 4f$ discussed above. Therefore, these DPA predictions are consistent with the results based on the full one-loop calculation. The DPA version presented in Ref. [8], however, deviates from the full one-loop calculation by about 0.9% for typical LEP2 energies.

In addition to the above results which include ISR beyond $\mathcal{O}(\alpha)$ and QCD corrections, we present also explicit numbers that include only the genuine $\mathcal{O}(\alpha)$ corrections to facilitate a comparison to calculations made by other groups in the future. Table 4 shows the relative $\mathcal{O}(\alpha)$ corrections, i.e. without higher-order ISR improvements and without QCD corrections, both for the DPA of RACOONWW and for the full $e^+e^- \rightarrow 4f$ calculation for the three final states $\nu_\tau\tau^+\mu^-\bar{\nu}_\mu$, $u\bar{d}\mu^-\bar{\nu}_\mu$, and $uds\bar{c}$.

3.3 Remaining theoretical uncertainties

We have reduced the TU for the charged-current processes $e^+e^- \rightarrow \nu_\tau\tau^+\mu^-\bar{\nu}_\mu$, $u\bar{d}\mu^-\bar{\nu}_\mu$, $uds\bar{c}$, in particular in the threshold region of W-pair production, considerably by calculating the full $\mathcal{O}(\alpha)$ corrections. ISR beyond $\mathcal{O}(\alpha)$ is included via structure functions in leading-logarithmic accuracy. For energies below ~ 500 GeV, the remaining uncertainties resulting from missing electroweak corrections are then dominated by the next-to-leading logarithmic electromagnetic corrections of order $(\alpha/\pi)^2 \log(m_e^2/s)$ which can be estimated to contribute $\lesssim 0.1\%$. Near the W-pair-production threshold, higher-order effects of the Coulomb singularity are still missing. These are estimated to $\sim 0.2\%$ [38, 39]. Thus, we estimate the theoretical uncertainty due to unknown electroweak higher-order effects in the present calculation to be a few 0.1% from the threshold region to about ~ 500 GeV. At higher energies leading and subleading electroweak high-energy logarithms, such as

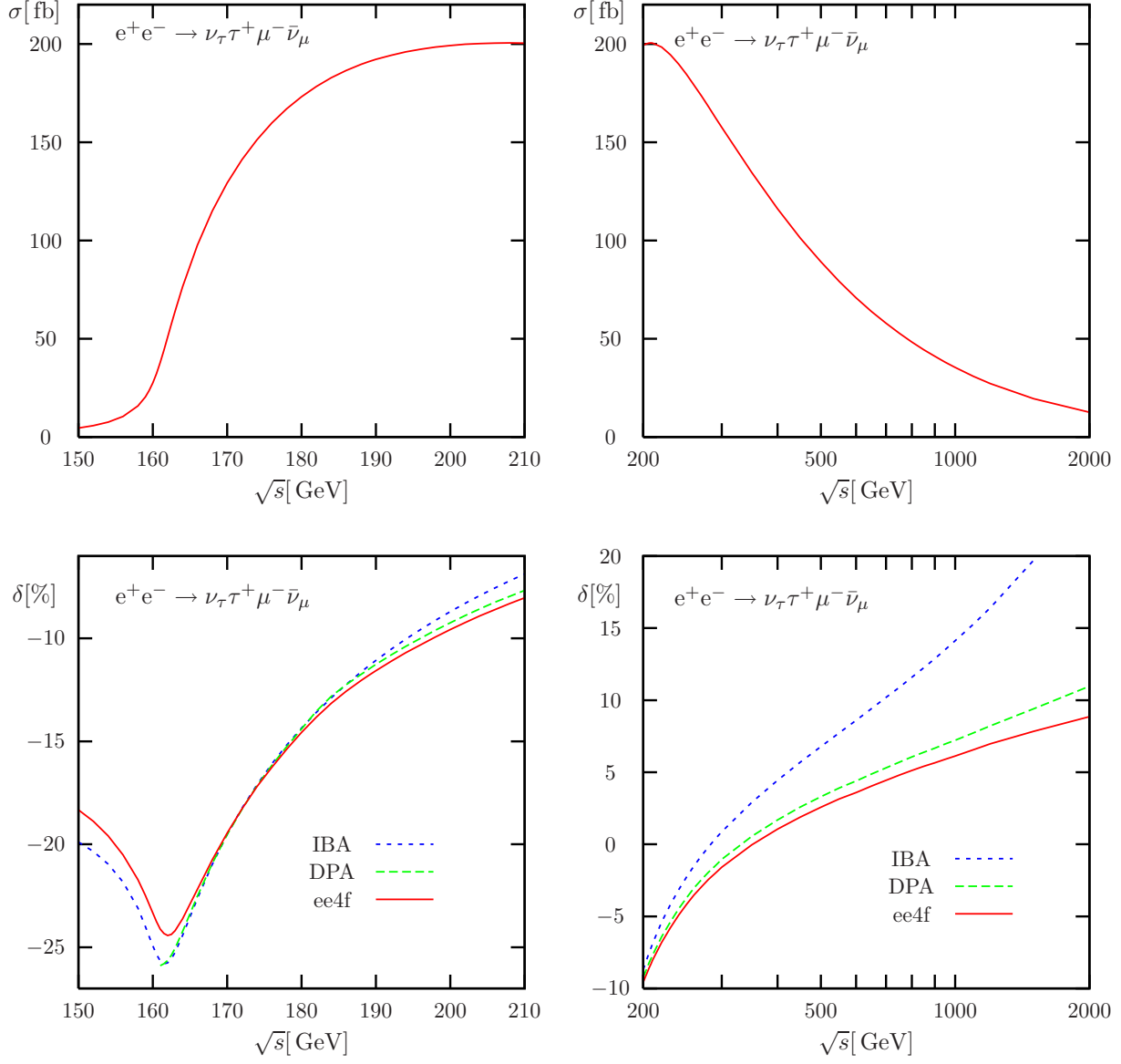


Figure 2: Absolute cross section σ (upper plots) and relative corrections δ (lower plots), as defined in the text, to the total cross section without cuts for $e^+e^- \rightarrow \nu_\tau \tau^+ \mu^- \bar{\nu}_\mu$ obtained from the IBA, DPA, and the full $\mathcal{O}(\alpha)$ calculation (ee4f). All predictions are improved by higher-order ISR.

$e^+e^- \rightarrow$	$\nu_\tau \tau^+ \mu^- \bar{\nu}_\mu$		$u\bar{d}\mu^- \bar{\nu}_\mu$		$u\bar{d}s\bar{c}$	
\sqrt{s}/GeV	$\delta_{\text{DPA}}[\%]$	$\delta_{\text{ee4f}}[\%]$	$\delta_{\text{DPA}}[\%]$	$\delta_{\text{ee4f}}[\%]$	$\delta_{\text{DPA}}[\%]$	$\delta_{\text{ee4f}}[\%]$
161	-31.30(3)	-29.53(3)	-31.30(3)	-29.56(3)	-31.27(3)	-29.62(3)
170	-21.67(2)	-21.59(2)	-21.75(2)	-21.65(2)	-21.81(2)	-21.72(2)
189	-11.74(2)	-12.05(2)	-11.84(2)	-12.13(2)	-11.94(2)	-12.22(2)
200	-9.15(2)	-9.48(2)	-9.25(2)	-9.57(2)	-9.33(2)	-9.64(2)
500	+3.55(3)	+2.84(4)	+3.50(3)	+2.76(4)	+3.37(3)	+2.63(4)
1000	+7.08(5)	+5.98(6)	+7.16(5)	+5.98(7)	+6.89(5)	+5.69(7)

Table 4: Genuine relative $\mathcal{O}(\alpha)$ corrections, i.e. without higher-order ISR improvements and QCD corrections.

Sudakov logarithms, beyond one loop have to be taken into account in addition to match this accuracy.

For a thorough estimate of the total theoretical uncertainty an inclusion of QCD effects is indispensable for the processes involving final-state quarks. In order to reach a precision of the order of a few 0.1% there, it is certainly necessary to improve the treatment of $\mathcal{O}(\alpha_s)$ corrections (and beyond), including a proper matching with parton showers. Bose–Einstein and colour interconnection effects may also play an important role.

4 Summary

We have presented results on total cross sections for the charged-current four-fermion production processes $e^+e^- \rightarrow \nu_\tau \tau^+ \mu^- \bar{\nu}_\mu$, $u\bar{d}\mu^- \bar{\nu}_\mu$, $u\bar{d}s\bar{c}$ which, for the first time, include the complete electroweak $\mathcal{O}(\alpha)$ corrections. The calculation is consistently performed using complex gauge-boson masses, supplemented by complex couplings to restore gauge invariance. The evaluation of the occurring one-loop tensor integrals, which include 5- and 6-point functions, required new techniques. Moreover, we have developed algorithms to reduce the large number of different spinor chains to a set of very few standard structures.

A comparison with the predictions for the total cross section without cuts based on the DPA provided by the generator RACOONWW reveals corrections beyond DPA of $\lesssim 0.5\%$ (0.7%) for CM energies ranging from $\sqrt{s} \sim 170 \text{ GeV}$ to 300 GeV (500 GeV), consistent with previous estimates on the intrinsic DPA uncertainty. The difference to the DPA increases to 1–2% for $\sqrt{s} \sim 1\text{--}2 \text{ TeV}$. At threshold, where predictions had to be based on an IBA at LEP2, the full $\mathcal{O}(\alpha)$ calculation corrects the IBA by about 1.6%, also consistent with a previous error estimate. The full $\mathcal{O}(\alpha)$ calculation, improved by higher-order effects from ISR, reduces the remaining TU due to unknown electroweak higher-order effects to a few 0.1% for scattering energies from the threshold region up to $\sim 500 \text{ GeV}$; above this energy leading high-energy logarithms, such as Sudakov logarithms, beyond one loop have to be taken into account to match this accuracy. At this level of accuracy, also improvements in the treatment of QCD corrections to semileptonic and hadronic $e^+e^- \rightarrow 4f$ processes will be necessary in the future.

Results for differential cross sections as well as details of the calculation will be presented elsewhere.

Acknowledgements

This work was supported in part by the Swiss National Science Foundation, by the Swiss Bundesamt für Bildung und Wissenschaft, and by the European Union under contract HPRN-CT-2000-00149.

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